Corrections to “Outage Analysis for Decode-and-Forward Multirelay Systems Allowing Intra-Link Errors”

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I. Introduction

In [1], we derived an analytical expression for the outage probability of decode-and-forward (DF) multirelay systems that allow for intra-link errors (IE). To this end, we relied upon the admissible rate region of a binary many-help-one problem with independently degraded helpers, which we believed to have obtained in [2]. Unfortunately, later on we realized that the admissible rate region in [2] is incorrect, and thus so is the resulting DF-IE outage probability in [1].

As far as we are aware, the admissible rate region of the binary many-help-one problem with independently degraded helpers remains unknown in closed form. On the other hand, we derived recently a simple bound of this admissible rate region when specialized to a primary source that is uniformly distributed and to helpers that are degraded through symmetric channels [3].

In this work we correct the analysis in [1] by obtaining an upper bound of the DF-IE outage probability based on the admissible rate region's bound in [3].

II. Corrections

The corrections that follow are threefold: the system model of DF-IE, the admissible rate region of the binary many-help-one problem with independently degraded helpers, and the outage probability of DF-IE. Please refer to the notation introduced in the last paragraph of [1, Section I], as we shall use it here.

System model: Unlike reported in [1, Section II] and depicted in [1, Fig. 1], the relay sequences are not decoded independently at the destination. Instead, all received sequences — from source and relays — are jointly decoded at the destination to retrieve the source message. In [2], we claimed that independent decoding would be able to achieve optimum performance. But we refuted this claim in [3].

Admissible rate region: In [1, Eqs. (2)–(3)], we reproduced from [2] what was claimed to be the admissible rate region of the binary many-help-one problem with independently degraded helpers. This admissible rate region turned out to be incorrect, and the problem remains open. On the other hand, we derived recently a bound on the admissible rate region when the primary source is uniformly distributed and the helpers are degraded through symmetric channels [3]. This bound is reproduced next, and used subsequently for outage analysis.

[3, Theorem 3]: *If $(X_1, X_2, ..., X_N)$ is an N-tuple of binary RVs with joint pmf $p(x_1, x_{\mathcal{L}}) = p(x_1)\prod_{i=2}^{N} p(x_i|x_1)$, with $p_{X_1}(0) = p_{X_1}(1) = 1/2$, $p_{X_i|X_1}(0|1) = p_{X_i|X_1}(1|0) = p_i$ for some $0 \leq p_i \leq 1/2, i \in \mathcal{L}$, then $\mathcal{R}_{sub}$ is a subset of the admissible rate region $\mathcal{R}_{DF\text{-}IE}$, given by*

$$\begin{aligned}\mathcal{R}_{\text{sub}} = \Big\{ &(R_1, R_2, ..., R_N): \\ &R_1 \geq \sum_{i\in\mathcal{L}} h\left(p_i * \kappa_i\right) - \eta(p_{\mathcal{L}}, \kappa_{\mathcal{L}}), \\ &\sum_{i\in\mathcal{S}} R_i \geq \eta(p_{\mathcal{L}}, \kappa_{\mathcal{L}}) - \eta(p_{\mathcal{S}^c}, \kappa_{\mathcal{S}^c}) - \sum_{i\in\mathcal{S}} h(\kappa_i), \\ &\sum_{i\in\mathcal{L}} R_i \geq 1 + \eta(p_{\mathcal{L}}, \kappa_{\mathcal{L}}) - \sum_{i\in\mathcal{L}} h(\kappa_i), \\ &\forall \mathcal{S} \subset \mathcal{L} \text{ and } \mathcal{S}^c = \mathcal{L}\backslash\mathcal{S}, \kappa_{\mathcal{L}} \in [0, 0.5]^{(N-1)} \Big\}, \end{aligned} \quad (1)$$

where $\eta(\cdot)$ is defined in [3, (27)]. *Here, we use a compact notation of its argument, e.g., $\eta(p_{\mathcal{L}}, \kappa_{\mathcal{L}}) = \eta(\{p_i * \kappa_i\}_{i\in\mathcal{L}})$.*

As shown in [3], the subset $\mathcal{R}_{\text{sub}}$ proves to be an increasingly tight approximation of the admissible rate region $\mathcal{R}_{\text{DF-IE}}$ as the helpers become more degraded.

Outage probability: In [1, Eqs. (6)–(9)], we claimed to have obtained the outage probability of DF-IE. But those expressions are incorrect, as they rely upon an incorrect admissible rate region [1, Eqs. (2)–(3)]. Here, by using (1), we obtain a valid upper bound on the referred outage probability, as follows. On the one hand, we have the transmission rates $R_{\mathcal{N}}$ depending on the received SNRs of the source- and relay-destination links, Γ_{SD} and $\Gamma_{F_{\mathcal{L}}D}$ (see [1, Eq. (4)]). On the other hand, we have the admissible rate region $\mathcal{R}_{\text{DF-IE}}$ depending on the received SNRs of the source-relay links $\Gamma_{SF_{\mathcal{L}}}$. An outage event occurs whenever the transmission rates $R_{\mathcal{N}}$ fall outside the admissible rate region $\mathcal{R}_{\text{DF-IE}}$. Thus, using [1, Eq. (4)], we have

$$\begin{aligned} P^{\text{out}}_{\text{DF-IE},N} = \Pr\Big[&\Big\{ \frac{1}{R_{\text{c}}}\phi(\Gamma_{SD}), \frac{1}{R_{\text{c}}}\phi(\Gamma_{F_{\mathcal{L}}D}) \Big\} \\ &\notin \mathcal{R}_{\text{DF-IE}}\left(\Gamma_{SF_{\mathcal{L}}}\right) \Big] \end{aligned} \quad (2)$$

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$$
\begin{aligned}
&< \Pr\Bigg[\bigg\{\frac{1}{R_\mathrm{c}}\psi(\Gamma_{SD}) \le \sum_{i\in\mathcal{L}} h\left(p_i(\Gamma_{SF_i}) * \kappa_l\right) \\
&\qquad\qquad\qquad - \eta(p_\mathcal{L}(\Gamma_{SF_\mathcal{L}}), \kappa_\mathcal{L})\bigg\} \cup \\
&\bigcup_{\forall \mathcal{S}\subset\mathcal{L}} \bigg\{\frac{1}{R_\mathrm{c}}\sum_{i\in\mathcal{S}}\psi(\Gamma_{F_iD}) \le \eta(p_\mathcal{L}(\Gamma_{SF_\mathcal{L}}), \kappa_\mathcal{L}) \\
&\qquad\qquad - \eta(p_{\mathcal{S}^\mathrm{c}}(\Gamma_{SF_{\mathcal{S}^\mathrm{c}}}), \kappa_{\mathcal{S}^\mathrm{c}}) - \sum_{i\in\mathcal{S}} h(\kappa_i)\bigg\} \cup \\
&\bigg\{\frac{1}{R_\mathrm{c}}\sum_{i\in\mathcal{L}}\psi(\Gamma_{F_iD}) \le 1 + \eta(p_\mathcal{L}(\Gamma_{SF_\mathcal{L}}), \kappa_\mathcal{L}) - \sum_{i\in\mathcal{L}} h(\kappa_i)\bigg\}, \\
&\mathcal{S}^\mathrm{c} = \mathcal{L}\backslash\mathcal{S}, \kappa_\mathcal{L} \in [0, 0.5]^{(N-1)}\Bigg]. \qquad (3)
\end{aligned}
$$

In (3), we substituted the rate constraints from (1). Because (1) represents a subset of the admissible rate region, the resulting probability is an upper bound on the outage probability of DF-IE. This bound cannot be solved in closed form, requiring numerical evaluation.

We verified for the numerical examples presented in [1, Fig. 2a] that the upper bound of DF-IE in (3) proves marginally different from the (incorrect!) outage probability given in [1, Eq. (9)]. The curves barely change. In particular, the conclusion drawn in [1] still holds true: DF-IE outperforms conventional DF in terms of outage probability, becoming more advantageous as more relays are used.

References

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[2] A. Wolf, D. C. González, M. Dörpinghaus, J. C. S. Santos Filho, and G. Fettweis, "On the many-help-one problem with independently degraded helpers," 2017. [Online]. Available: http://arxiv.org/abs/1701.06416v1

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